Kufa Journal of Engineering
Vol. 10, No. 2, 2019, P.P. 27-41
Received 14 November 2017, accepted 23 April 2018

IMAGE COMPRESSION BY USING WALSH AND FRAMELET TRANSFORM

Nora H. Sultan[1]

[1]**Asst. Lecturer, Department of Computer, Collage of Education for Women, University of Kufa, Iraq. Email: norah.mayali@uokufa.edu.iq**

http://dx.doi.org/10.30572/2018/kje/100203

ABSTRACT

In this paper, Framelet and Walsh transform are proposed for transformation, and then using arithmetic coding for compress an image. The goal is to achieve higher compression ratio by applying two levels Framelet transform (FLT), and then apply 2D Walsh-Hadamard transform (WHT) on each 8x8 block of the low frequency sub-band, while all other sub-bands are ignored. Experimental results show that the proposed algorithm gets best possible solution for tradeoff between compression ratio (size of image) and quality of compressed image, Peak Signal to Noise Ratio (PSNR). The simulation was carried using MATLAB software package version 2014. In this work, experiments were carried out on the gray scale and colored images

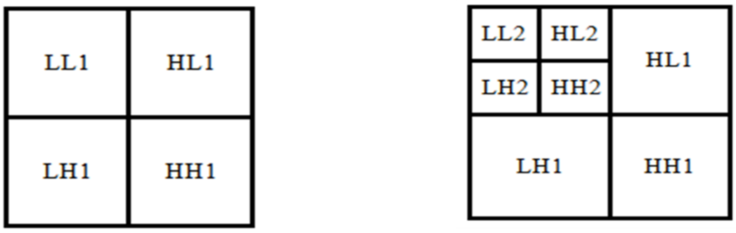
KEYWORDS: Walsh Hadamard Transform (WHT); Framelet Transform (FLT); Peak Signal to Noise Ratio (PSNR); Compression Ratio (CR); Arithmetic coding.

1. INTRODUCTION

Image compression is an inevitable solution for image transmission since the channel bandwidth is limited and the demand is for faster transmission (Alsayyh and Dzulkifli, 2012). The two major types of compression algorithms: lossless compression and lossy compression. Lossless compression is used for applications that require an exact reconstruction of the original data, while lossy compression is used when the user can tolerate some differences between the original and reconstructed representations of the data. In return for accepting this distortion in the reconstruction, we can generally obtain much higher compression ratios than is possible with lossless compression (Khalid Sayood, 2006). Transform based image coding is one of the popular image compression method. Transform when applied on images; change the image pixels to frequency domain coefficients. Desirable property of transforms is that most of the image energy is concentrated only in few significant transform coefficients. Retaining these significant coefficients and eliminating remaining coefficients results in image compression. Discrete Cosine Transform (DCT) and wavelet transform are commonly used transform methods for image compression. They are used in JPEG and JPEG 2000 respectively (Kekre et al., 2014). Recent advancements in this area show that transform based coding combined with other compression method results in better performance. In this paper, Framelet transform is used with Walsh transform for image compression. Framelet is very similar to wavelets, but has some important differences. Framelet has two or more high frequency filter banks, which produces more sub bands in decomposition. This can achieve better time- frequency localization ability in signal processing. Moreover, Framelet is more robust (Jiao and Lin, 2010). And the Walsh- Hadamard transform (WHT) is an invertible linear transform and is widely used in many practical image compression systems because of its compression performance and computational efficiency (Thida and Aye, 2015). The elements of the basis vectors of the Hadamard Transform take only the binary values ±1 and are, therefore, well suited for digital hardware implementations of image processing algorithms. Hadamard transform offers a significant advantage in terms of a shorter processing time as the processing involves simpler integer manipulation (compared to floating point processing with DCT) and the ease of hardware implementation than many common transform techniques. So it is computationally less expensive than many other orthogonal transforms (Veeraswamy and Srinivaskumar, 2017). In this work, will be applying 2D Walsh-Hadamard transform on each 8x8 block of the low frequency sub-band of Framelet transform, while all other sub-bands are ignored to get higher compression ratio. Such method is lossy compression method, then entropy coding of the quantized coefficients by Arithmetic coding (Arithmetic coding gives greater compression than Huffman method, is faster for adaptive models, and clearly separates the model from the channel encoding (Moffat, et al, 1998). Quantization is the process of reducing the number of possible values of a quantity, thereby reducing the number of bits needed to represent it. Quantization is a lossy process and implies in a reduction of the color information associated with each pixel in the image (Thida and Aye, 2015).

2. DISCRETE WAVELET TRANSFORM (DWT)

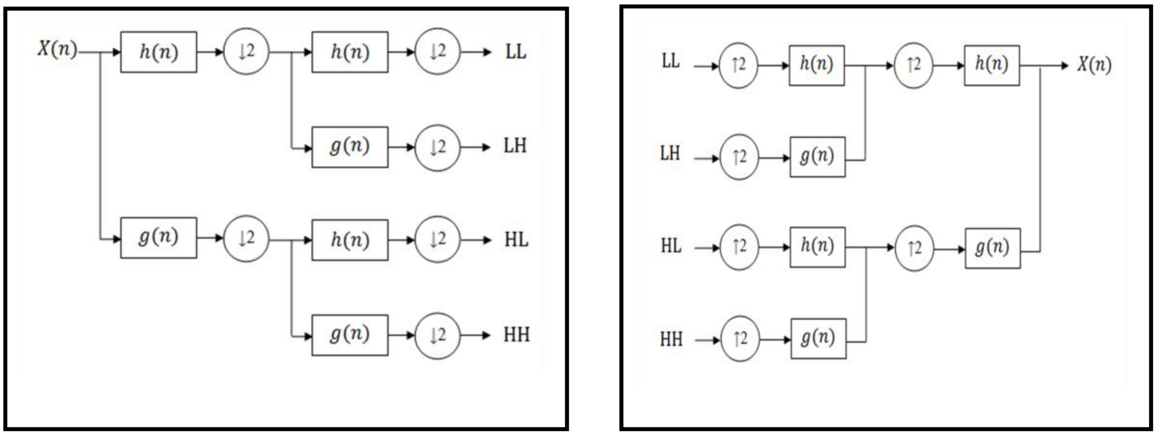
In DWT, image is divided into four sub bands as shown in Fig. 1 a. These sub bands are formed by separable applications of horizontal and vertical filters. Coefficients that are represented as sub bands LH1, HL1 and HH1 are detail images while coefficients are represented as sub band LL1 is approximation image. The LL1 sub band is further decomposed to obtain the next level of wavelet coefficients as shown in Fig. 1b (Kumar and Agarwal, 2015).



(a) One level **(b) Two level**

Fig. 1. Decomposition DWT.

Fig. 2 (a and b) show that the filter bank structure for (2D analysis DWT and 2D synthesis DWT), respectively.



(a) Analysis **(b) Synthesis**

Fig. 2. Filter bank for 2D- DWT Analysis and Synthesis (Barni, 2004).

3. FRAMELET TRANSFORM (FLT)

The three-channel filter bank, which is used to develop the FLT corresponding to a wavelet frame based on a single scaling function $\phi(t)$ and two distinct wavelets $\Psi_1(\mathrm{t})$ and $\Psi_2(\mathrm{t})$ the extra wavelet here makes this system an over complete one. It follows that $\phi(t)$, $\Psi_1(\mathrm{t})$ and $\Psi_2(\mathrm{t})$ satisfies the dilation and wavelet equations (Choi, 2007).

$$\phi(t) = \sqrt{2}\sum_n \mathrm{h}_0(n)\,\phi(2t-n) \qquad 1$$

$$\Psi_i = \sqrt{2}\sum_n \mathrm{h}_i(n)\,\phi(2t-n), \qquad i = 1,2 \qquad 2$$

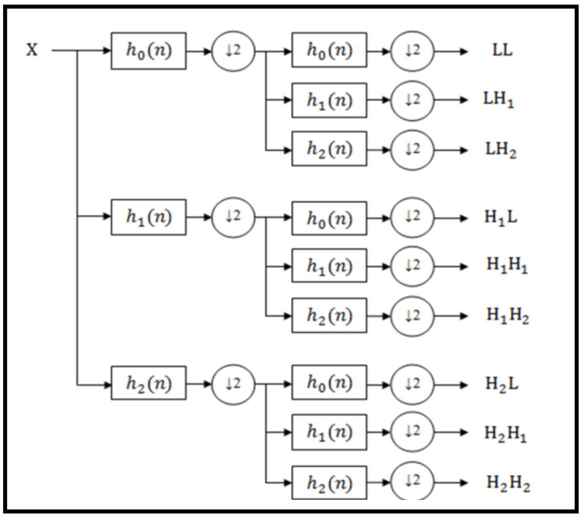
The scaling function $\phi(t)$ and the wavelets $\Psi_1(\mathrm{t})$, $\Psi_2(\mathrm{t})$ are defined through these equations by the low-pass (scaling) filter $\mathrm{h}_0(n)$ and the two high-pass (wavelet) filters $\mathrm{h}_1(n)$ and $\mathrm{h}_2(n)$, where the two distinct wavelets $\Psi_1(\mathrm{t})$ and $\Psi_2(\mathrm{t})$ are specifically designed to be offset from one another by one half as follows:

$$\Psi_1(t) \cong \Psi_2(t-0.5) \qquad 3$$

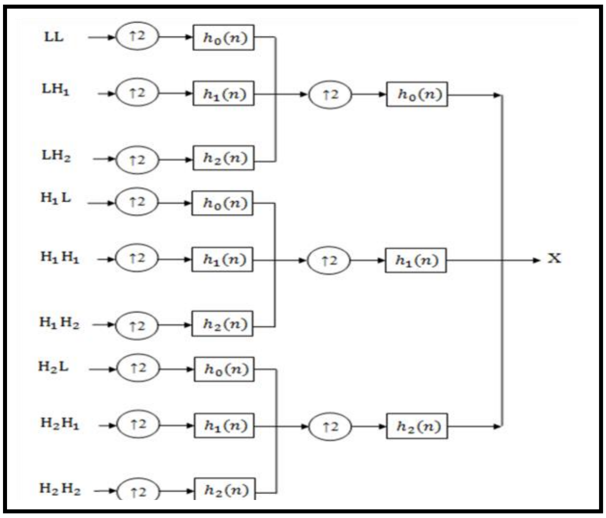
Where the filters $\mathrm{h}_1(n)$ and $\mathrm{h}_2(n)$ should satisfy the Perfect Reconstruction (PR) condition. This means that the input and output of the two filters are expected to be the same (Choi, 2007).

3.1. Filter bank structure for 2D framelet

To perform the FLT on 2D matrix, the transform first should be alternatively applied to the rows, then to the columns of the resulting matrices. This gives rise to nine 2-D sub-bands, one of which is the 2-D low pass scaling filter, and the other eight of which make up the eight 2-D wavelet filters, as shown in Fig. 3 (Abdulkareem, 2012).



(a) Analysis



(b) Synthesis

Fig. 3. Analysis and synthesis Stages of a 2-D Single Level FLT.

3.2. computational of FLT for 2-D signal using separable method

A separable 2-D FLT can be obtained by alternating between rows and columns i.e., it processes each row in order and then processes each column of the result. Non-separable methods work in both matrix dimensions at the same time. A 2-D separable transform is equivalent to two 1-D transforms in series. It is implemented as 1-D row transform followed by a 1-D column transform on the data obtained from the row transform. To compute a single level discrete framelet transform for 2-D signal using separable method, the next steps should be followed (Abdulkareem, 2012):

- Checking dimensions: Input matrix should be of size $N \times N$, where N must be even and $N \geq$ length (analysis filters).
- Construct a transformation matrix: For an $N \times N$ matrix input 2-D signal X , construct a $\frac{3N}{2} \times N$ transformation matrix, W , using length-7 coefficients filter (for example) given as (Abdulkareem, 2012):

$$W = \begin{bmatrix} h_0(0) & h_0(1) & h_0(2) & h_0(3) & h_0(4) & h_0(5) & h_0(6) & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & h_0(0) & h_0(1) & h_0(2) & h_0(3) & h_0(4) & h_0(5) & h_0(6) & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ h_0(2) & h_0(3) & h_0(4) & h_0(5) & h_0(6) & 0 & 0 & \dots & 0 & 0 & h_0(0) & h_0(1) \\ h_1(0) & h_1(1) & h_1(2) & h_1(3) & h_1(4) & h_1(5) & h_1(6) & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & h_1(0) & h_1(1) & h_1(2) & h_1(3) & h_1(4) & h_1(5) & h_1(6) & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ h_1(2) & h_1(3) & h_1(4) & h_1(5) & h_1(6) & 0 & 0 & 0 & 0 & \dots & h_1(0) & h_1(1) \\ h_2(0) & h_2(1) & h_2(2) & h_2(3) & h_2(4) & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & h_2(0) & h_2(1) & h_2(2) & h_2(3) & h_2(4) & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ h_2(2) & h_2(3) & h_2(4) & 0 & 0 & 0 & 0 & 0 & 0 & \dots & h_2(0) & h_2(1) \end{bmatrix}_{\frac{3N}{2} \times N}$$

- Transformation of input rows by applying matrix multiplication to the $\frac{3N}{2} \times N$ constructed transformation matrix by the $N \times N$ input matrix.

$$Y = [W]_{\frac{3N}{2} \times N} \times [X]_{N \times N} \qquad 4$$

- Transformation of input columns: can be done as follows (Abdulkareem, 2012):
  - Transpose the row transformed $\frac{3N}{2} \times N$ matrix resulting from step (3).
  - Apply matrix multiplication to the $\frac{3N}{2} \times N$ constructed transformation matrix by the $N \times N$ column matrix.

$$YY = [W]_{\frac{3N}{2}\times N} \times [Y]^{T}{}_{N\times\frac{3N}{N}} \quad 5$$

The final framelet transformed matrix is equal to:

$$Y_0 = [YY]^{T}{}_{\frac{3N}{2}\times\frac{3N}{2}} \quad 6$$

3.3. Computation of IFLT for 2-D signal using separable and non-separable methods

To reconstruct the original signal from the discrete framelet transformed signal, Inverse Framelet Transform (IFLT) should be used. The inverse transformation matrix is the transpose of the transformation matrix as the transform is orthogonal, so to compute a single level 2-D inverse discrete framelet transform using separable method, the next steps should be followed (Abdulkareem, 2012; Qasim, 2010):

- Let Y_0 be $\frac{3N}{2} \times \frac{3N}{2}$ framelet transformed matrix.
- Construct $N \times \frac{3N}{2}$ reconstruction matrix, using transpose of transformation matrix, $[W]^T$.
- Reconstruction of columns: by applying matrix multiplication to the $N \times \frac{3N}{2}$ reconstruction matrix by the $\frac{3N}{2} \times \frac{3N}{2}$ framelet transformed matrix.

$$YYX = [W]^{T}{}_{N\times\frac{3N}{2}} \times [Y_0]_{\frac{3N}{2}\times\frac{3N}{2}} \quad 7$$

- Reconstruction of rows: can be done as follows:
  - Transpose the column reconstructed matrix resulting from step (3).

$$YX = [YYX]^{T}{}_{\frac{3N}{2}\times N} \quad 8$$

  - b) Apply matrix multiplication by multiplying the reconstruction matrix with the resultant transpose matrix.

$$XX = [W]^{T}{}_{N\times\frac{3N}{2}} \times [YX]_{\frac{3N}{2}\times N} \quad 9$$

$$X = [XX]^T \quad 10$$

To compute a single level inverse framelet transform for 2-D signal using non-separable method, the next steps should be followed:

- Let Y_0 be $\frac{3N}{2} \times \frac{3N}{2}$ framelet transformed matrix.
- Construct $N \times \frac{3N}{2}$ reconstruction matrix W^T using transformation matrix.

- Reconstruction of the input matrix by multiplying the reconstruction matrix by the input matrix and by the transpose of the reconstruction matrix.

$$X = W^T \times Y_0 \times W \quad (11)$$

A single-stage framelet transformation consists of nine frequency bands. These subbands are shown in Fig. 4. Since L is a low-pass filter $h_0(n)$ while both H_1 and H_2 are high-pass filters ($h_1(n)$ and $h_2(n)$) , the $H_1\,H_1$, $H_2\,H_1$, $H_1\,H_2$ and $H_2\,H_2$ subbands each has a frequency-dom ain support comparable to that of the HH subband in a DWT. A similar scheme creates the $H_1\,L, H_2\,L, LH_1$ and LH_2 subbands with the same frequency-domain support as the corresponding $HL(LH)$ subbands of the DWT, but with twice as many coefficients. Finally, note that there is only one subbands LL with the same frequency-domain as the LL subbands in a DWT (Abdulkareem, 2012).

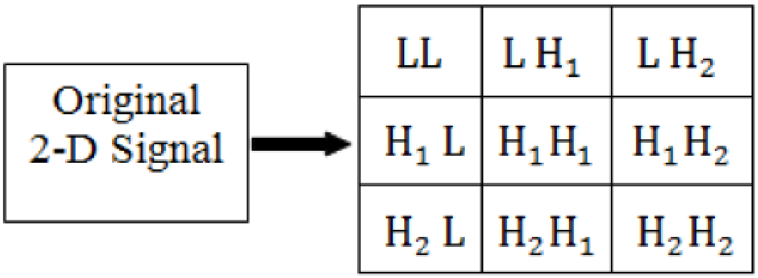


Fig. 4. Matrix subbands after a single-level decomposition FLT.

4. WALSH–HADAMARD TRANSFORM (WHT)

Discrete Fourier transform (DFT), discrete cosine transform (DCT), and Walsh–Hadamard transform (WHT) are widely used in the image processing applications. These linear image transforms are chosen in the image processing application because of their flexibility, energy compaction, and robustness. These transforms effectively extract the edges and also provide energy compaction in the state-of-the-art methods. Among all these transforms, WHT is very gorgeous one because of its simplicity and its computational efficiency. The major properties of WHT are same as that of other image transforms. The basis vector components of WHT are orthogonal, and it is having binary values (±1) only (Verma and Kadian, 2014).

The discrete Walsh-Hadamard transform matrices are rearrangements of discrete Hadamard matrices, which are of particular importance in coding theory. A Hadamard matrix of order N is defined as an N ×N matrix H, with the property that $HH^T = NI$, where I is the N ×N identity matrix. Hadamard matrices whose dimensions are a power of two can be constructed in the following manner (Khalid Sayood, 2006):

$$H_{2N} = \begin{bmatrix} H_N & H_N \\ H_N & -H_N \end{bmatrix}$$

with H1 = [1]. Therefore,

$$H_2 = \begin{bmatrix} H_1 & H_1 \\ H_1 & -H_1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$H_4 = \begin{bmatrix} H_2 & H_2 \\ H_2 & -H_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

$$H_8 = \begin{bmatrix} H_4 & H_4 \\ H_4 & -H_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix}$$

The DWHT transform matrix H can be obtained from the Hadamard matrix by multiplying it by a normalizing factor so that HH^T = I instead of NI, and by reordering the rows in increasing sequency order. The sequency of a row is half the number of sign changes in that row. In H_8 the first row has sequency 0, the second row has sequency 7/2, the third row has sequency 3/2, and so on. Normalization involves multiplying the matrix by $1/\sqrt{N}$. Reordering the H_8 matrix in increasing sequency order, we get:

$$H = \frac{1}{\sqrt{8}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \end{bmatrix}$$

Because the matrix without the scaling factor consists of ±1, the transform operation consists simply of addition and subtraction. For this reason, this transform is useful in situations where minimizing the amount of computations is very important.

5. PROPOSED ALGORITHM

Fig. 5 shows the flow chart of proposed algorithm:

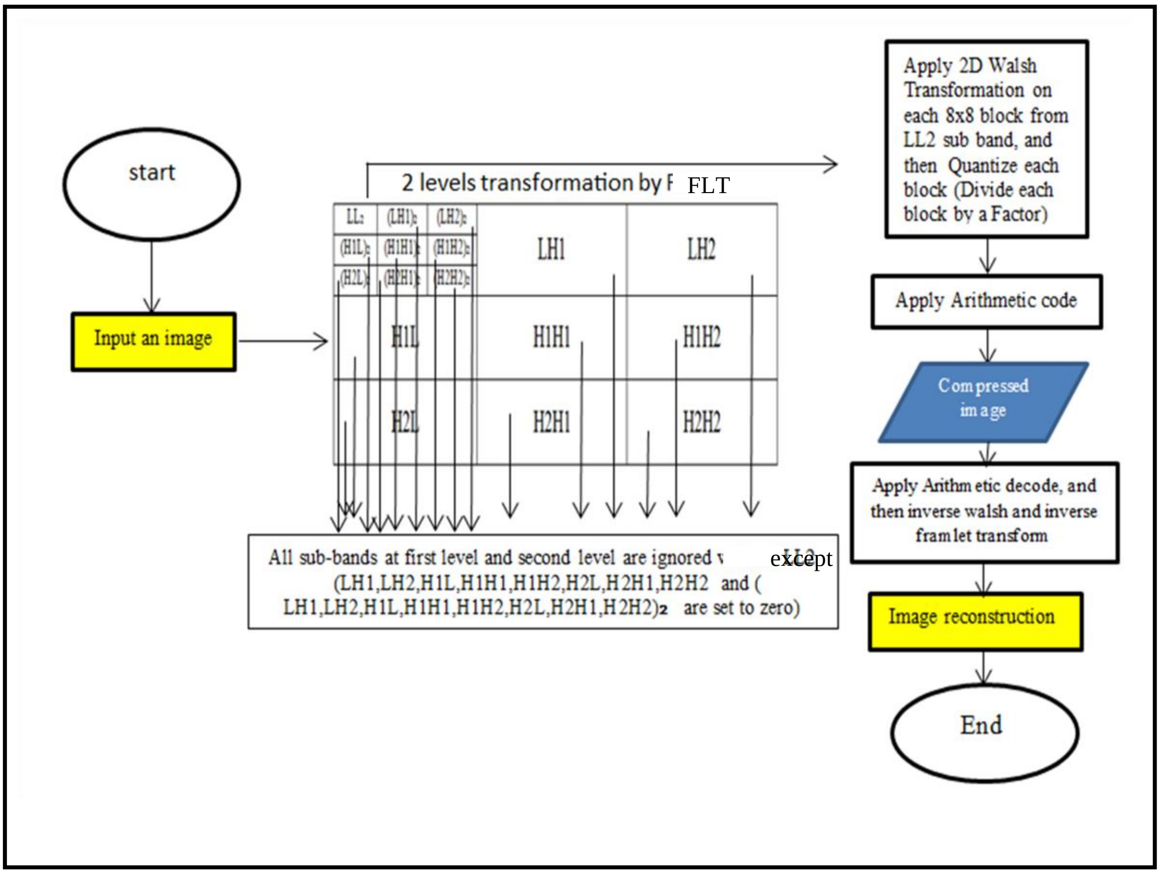


Fig. 5. Flow chart of proposed algorithm.

In this work, 2-Level 2-Dframelet decomposition is applied on the original image, then apply Walsh transformation on low frequency domain and ignore all high frequency domains. Then apply arithmetic code. The decompressed image is reconstructed by applying inverse Walsh transform and inverse framelet transform.

6. COMPRESS AND DECOMPRESS COLOR IMAGES

This algorithm is proposed for compressing the color images. First the RGB colors images are converted into $Y\,C_bC_r$ form, then applying proposed algorithm on each layer independently, this means each layer from $Y\,C_bC_r$ are compressed as a gray scale image. Fig. 6 shows that proposed algorithm is applied on each $Y\,C_bC_r$ layer.

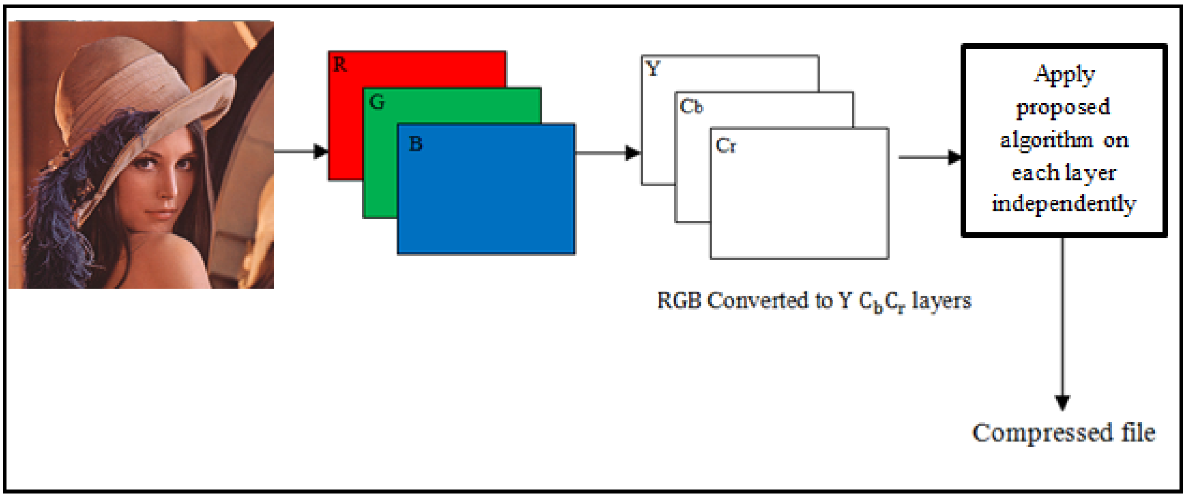


Fig. 6: RGB Layers are Converted to YC_bC_r Layer, and then Compressed by proposed Algorithm.

For decompression color images, apply decompression on each layer then collect all layers in one matrix Y C_bC_r and convert Y C_bC_r format to RGB color image.

7. RESULTS AND DISCUSSION

Fig. 7 shown some gray scale images with proposed algorithm and the results in Tables 1, 2, 3 and 4 shown the performance of PSNR and CR compared with other algorithms

Table 1. Performance comparison between proposed algorithm and other algorithms for lena image.

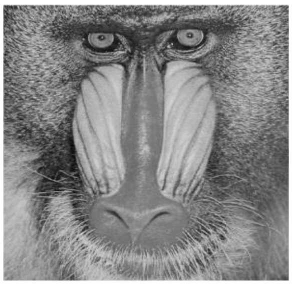
Algorithm	CR	PSNR	Time (s)
Proposed algorithm using Walsh and discrete wavelet transform (db1)	26.1542	26.0871	2.3585
Proposed algorithm using Walsh and discrete wavelet transform (db3)	22.826	29.1551	2.6486
Proposed algorithm using Walsh and discrete wavelet transform (db5)	23.4371	29.4685	2.657
Proposed algorithm using Walsh and Framelet transform	26.0762	27.6329	2.6234
Efficient image compression using artificial neural network (Verma and Kadian, 2014)	1.75	26.87	81.42

(a) Original image Lena.bmp

(b) Decompressed Lena

(a) Original image Barbara.bmp

(b) Decompressed Barbara



(a) Original image mandrill.tiff

(b) Decompressed mandrill

(a) Original image eye.bmp

(b) Decompressed eye

Fig. 7. Comparision between Original Image and Decompressed Image.

Table 1 illustrates that the proposed algorithm using Walsh and framelet transform is the best choice for get higher compression ratio with good quality for image by compared with the other algorithms.

Table 2. PSNR and Time values for 26:1 compression for lena image in different algorithms.

Algorithm	PSNR	Time (s)
Hybrid DCT-VQ (Rehna and Kumar, 2011)	25.3996	43.35
Modified Levenberg-Marquardt Method (Prena, 2011)	22.3675	2169.579
Proposed algorithm using Walsh and Framelet transform	27.6329	2.6234

From the results tabulated in Table 2, the time taken in proposed algorithm is lower than in other algorithms and gives higher PSNR

Table 3. Performance comparison between proposed algorithms at barbara image.

Algorithm	CR	PSNR	Time (s)
Proposed algorithm using Walsh and discrete wavelet transform (db1	25.1216	21.5519	2.5467
Proposed algorithm using Walsh and discrete wavelet transform (db3)	21.8472	22.4649	2.8162
Proposed algorithm using Walsh and discrete wavelet transform (db5)	22.6553	22.7160	2.6952
Proposed algorithm using Walsh and Framelet transform	25.5850	21.9662	2.6177

Table 4. Performance comparison between proposed algorithms at eye image.

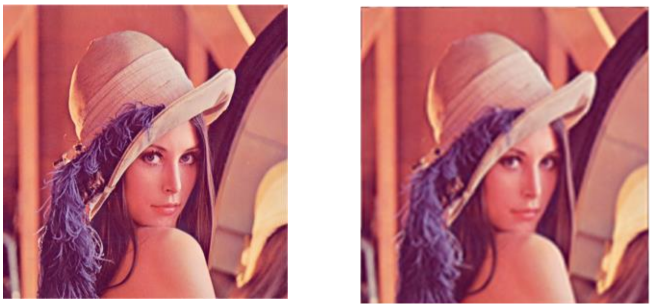
Algorithm	CR	PSNR	Time (s)
Proposed algorithm using Walsh and discrete wavelet transform (db1)	31.1705	30.4274	2.2451
Proposed algorithm using Walsh and discrete wavelet transform (db3)	27.4181	32.0927	2.4667
Proposed algorithm using Walsh and discrete wavelet transform (db5)	28.4167	32.2037	2.4154
Proposed algorithm using Walsh and Framelet transform	31.8716	31.3748	2.3854

Table 5. Performance comparison between proposed algorithms at mandrill image.

Algorithm	CR	PSNR	Time (s)
Proposed algorithm using Walsh and discrete wavelet transform (db1)	23.7859	19.1179	2.6967
Proposed algorithm using Walsh and discrete wavelet transform (db3)	21.0541	20.5527	2.9726
Proposed algorithm using Walsh and discrete wavelet transform (db5)	20.8348	19.9292	2.9293
Proposed algorithm using Walsh and Framelet transform	24.5937	19.2109	2.8851

The above comparison of Barbara & eye & mandrill images in Tables 3, 4 and 5, respectively, show that the proposed algorithm using Walsh and framelet transform gives the best results for compression ratio with maintaining good quality for images.

Finally testing the proposed algorithm on Lena colour image. The comparison between original and decompressed image is shown in Fig. 8:



(c) Original image Lena.bmp (b) Decompressed Lena

Fig. 8. Comparison between Original Colour Test Image and its Decompressed Image.

Good image quality and good compression ratio for the color images are illustrate in Table 6 after applying proposed algorithm.

Table 6: Color Images of the Proposed Algorithm.

Algorithm	CR	PSNR	Time (s)
Proposed algorithm using Walsh and discrete wavelet transform (db1)	12.1737	27.0997	5.0602
Proposed algorithm using Walsh and discrete wavelet transform (db3)	12.6186	29.4078	5.3341
Proposed algorithm using Walsh and discrete wavelet transform (db5)	11.1592	29.7541	5.5396
Proposed algorithm using Walsh and Framelet transform	12.5288	28.4541	5.2383

8. CONCLUSION

The evaluation of performance using some objective criteria such as CR and PSNR show that the proposed algorithm based on WHT with FLT provides an important compression ratio while keeping a good quality of reconstructed image. A comparative study between the proposed algorithm and the algorithm with DWT. The properties of Framelet transform helped the algorithm to get good image quality, where the main reason for using Framelet transform is to reduce the image dimensions, while maintaining image quality. So, the high-frequencies coefficients are ignored.

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